Using Field Theory to Measure Surface Resistivity of High-Resistance Polymeric Films

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ABSTRACT: Theoretical expressions based on Ohm's law for field theory were derived to measure surface resistivity of high-resistance polymer films using parallel plate and concentric circular electrodes. Experimental measurements of resistivity were compared to experimental measurements using conventional expressions based on Ohm's law for circuit theory. Expressions based on Ohm's law for circuit theory were found to be inadequate for measuring surface resistivity. The surface resistivity of high-resistance films can be accurately measured if electric field theory is used to include the effects of electrode structure, if relative humidity (RH) and temperature are controlled, if the level of applied voltage is limited, and if the measuring system is shielded from extraneous electric and magnetic fields. © 2001 John Wiley & Sons, Inc. J Appl Polym Sci 82: 2856–2862, 2001

Key words: surface resistivity; high-resistance films; electric field theory; Ohm's law; electrodes

INTRODUCTION

Electrical properties are important for many applications of materials and numerous analytical techniques have been developed to characterize application-specific electrical properties. These include assessing the propensity of a fabric to pick up dust, measuring the propensity of a charged material to ignite a gaseous mixture with an electrostatic discharge, measuring the voltage produced on a person's body while walking on a carpet, and measuring the cling time of apparel fabrics.

In addition, numerous analytical techniques have been developed to characterize more fundamental electrical properties of materials. These include measuring surface resistivity, volume resistivity, charge generation ability, charge decay rate, and spark discharge energy.

Surface resistivity is a measure of the static behavior on high-resistance materials. ASTM D257[1] is a standard test method to measure the surface and volume resistivities of insulating materials using the theory of Ohm's law for circuit theory. Unfortunately, Burdeaus et al.[2] showed that surface resistivity measured by this ASTM test method varied by a factor of 2–3 when different electrode diameters were used. Similar results have been observed in our research using two parallel square electrodes placed at different separation distances.

In this article, we address the measurement of surface resistivity using Ohm's law for field theory and compare it to measurements using Ohm's law for circuit theory. We also control the electrode configuration and other parameters which

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affect resistivity measurements, including relative humidity (RH), temperature, electric and magnetic field interference, and applied voltage.

THEORY

According to Ohm's law for circuit theory, the electric resistance (R) of a material can be calculated from the applied voltage (V) and current (I) drawn across the material between two electrodes

$$R = \frac{V}{I} \tag{1}$$

The electrical resistance of a sample is proportional to its length (l) and inversely proportional to its cross-sectional area (A)

$$R \propto \frac{l}{A} \tag{2}$$

A constant ρ can be introduced into Eq. (2) to obtain

$$R = \rho \frac{l}{A} \tag{3}$$

The constant ρ is resistivity, a fundamental electrical property of a material that is independent of the sample dimension and shape. The electrical resistance of a sample is characterized by a fundamental material property (ρ) and the geometric structure of the test sample (l/A).

If the test sample has a rectangular shape with unit thickness (t), Eq. (3) can be written as

$$\rho = R\frac{wt}{l} = R\frac{w}{l} \tag{4}$$

where w is the width of the sample.

The resistance of a square sample can be considered by analogy with an electric circuit to be a resistor with resistance Ro, as shown in Figure 1(a). According to Eq. (4), its resistivity (ρ) equals its resistance Ro because $w = l$. The resistance of a rectangular sample with length twice its width will be 2Ro, according to Eq. (4), and can be considered by analogy to be two resistors Ro connected in series, as shown in Figure 1(b). However, its resistivity is still expected to be Ro because resistivity is a fundamental material property which is independent of test sample dimensions. Similarly, the resistance of a rectangular sample with width twice its length will be $\frac{1}{2}$ Ro and can be considered by analogy to be two resistors Ro connected in parallel, as shown in Figure 1(c). Its resistivity is still expected to be Ro.

These examples illustrate that the resistivity of a test sample with unit thickness is expected to equal the resistance of the sample in square dimension regardless of its in-plane dimensional size. Surface resistivity is the resistivity of a material with unit thickness and the unit of surface resistivity is ohm/square.

The resistivity of a circular test sample with unit thickness (Fig. 2)can be derived starting with Ohm's law for circuit theory in the same way to derive resistivity for the rectangular sample:

$$\begin{aligned} R &= \rho\frac{w}{l} \\ &= \rho \lim_{\Delta r \to 0} \sum_{n=1}^{\infty} \frac{\Delta r}{2\pi(a + n\Delta r)} \\ &= \rho \int_a^b \frac{dr}{2\pi r} \\ &= \frac{\rho}{2\pi} \ln\left(\frac{b}{a}\right) \end{aligned}$$

So

$$\rho = 2\pi \frac{R}{\ln\left(\frac{b}{a}\right)} \tag{5}$$

where a equals the diameter of the inner electrode and b equals the diameter of the outer electrode. Equation (5) provides the surface resistivity for a test sample measured using two concentric circular electrodes.

Equations (4) and (5) were derived for rectangular and circular samples using Ohm's law for circuit theory. In addition, Ohm's law only provides an approximate way to calculate resistance and may contain substantial error for high-resistance and irregular-shape materials.[3,4] High-resistance materials draw little current and require Ohm's law for electric field theory to calculate resistance accurately.

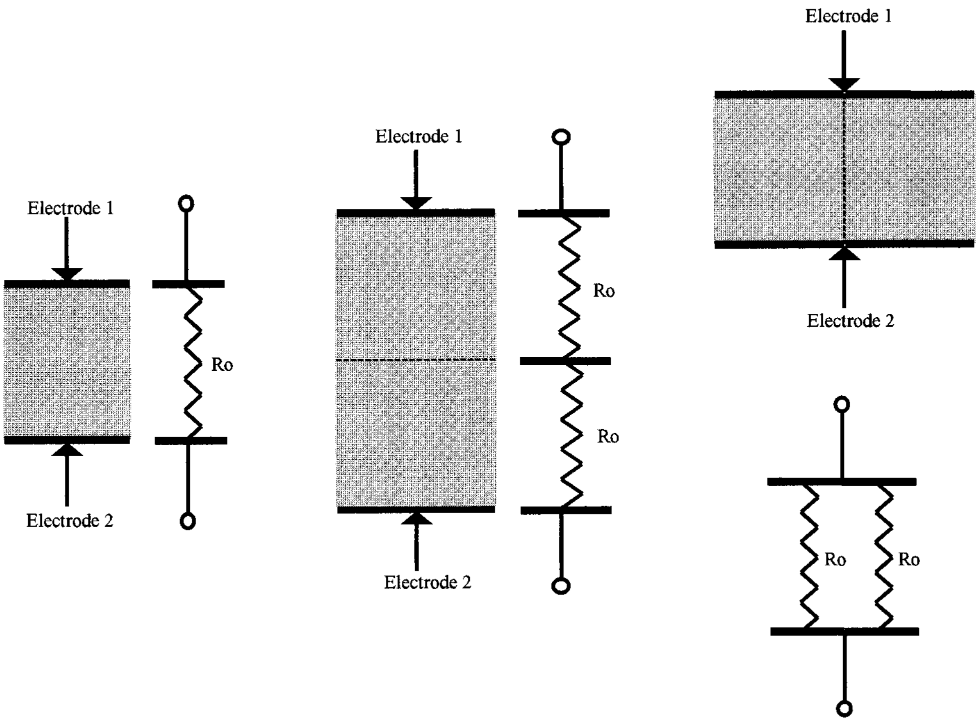

Figure 1 Square sample as (a) single resistor (left), (b) two resistors in series (center) and (c) two resistors in parallel (right).

Ohm's law can be stated in point form or for field theory so current density (J) in the sample is related to the electric field (E) provided by the electrodes and by the conductivity of the sample (σ)

$$\vec{J} = \sigma \vec{E} \tag{6}$$

If the electrodes are considered to be two infinite parallel plates, the electric field between them is unidirectional and can be expressed as V/d, where d is the distance between the plates. Thus,

$$\vec{J} = \frac{1}{\rho}\frac{V}{d}\hat{E} \tag{7}$$

Current moving across a sample between two parallel plates can be obtained by integrating current density over the sample cross-sectional area, A

$$I = \int_A \vec{J} \cdot d\vec{A}$$

$$= \frac{1}{\rho}\frac{V}{d} w$$

So

$$\rho = \frac{V}{I}\frac{w}{d} \tag{8}$$

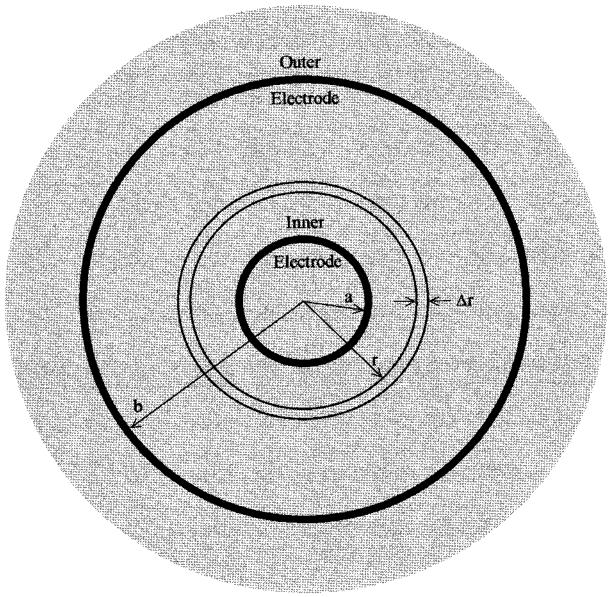

Figure 2 Circular sample for surface resistivity measurements.

Two parallel plates were specified as electrodes when deriving Eq. (8) using Ohm's law for field theory.

Similarly, current moving across a sample between two concentric cylindrical electrodes can be derived using Ohm's law in point form by considering the electric field distribution ($E(r)$) between the two electrodes

$$\vec{E}(r) = \frac{V}{r\ln\left(\frac{b}{a}\right)}\hat{r} \tag{9}$$

The current density is

$$\vec{J}(r) = \sigma\vec{E}(r) = \frac{1}{\rho}\frac{V}{r\ln\left(\frac{b}{a}\right)}\hat{r} \tag{10}$$

and the current is

$$\begin{aligned} I &= \int_A \vec{J}\cdot d\vec{A} \\ &= \frac{1}{\rho}\frac{2\pi V}{\ln\left(\frac{b}{a}\right)} \end{aligned} \tag{11}$$

Therefore, the surface resistivity is

$$\rho = \frac{2\pi}{\ln\left(\frac{b}{a}\right)}\frac{V}{I} \tag{12}$$

Two cylindrical electrodes were specified when deriving Eq. (12) using Ohm's law in point form or for field theory.

EXPERIMENTAL

Two types of electrodes were constructed to measure surface resistivity using Ohm's law for field theory. Cylindrical electrodes were constructed as illustrated in Figure 2 with outer electrode diameters of 4.02 cm and 6.29 cm and an inner electrode diameter of 0.64 cm. Four L-shaped parallel plates were constructed as shown in Figure 3. Width of the plates equals 15.2 cm and height equals 10.2 cm. The separation distance between the two plates was 2.54 cm and 5.1 cm and the sample width was 10.2 cm.

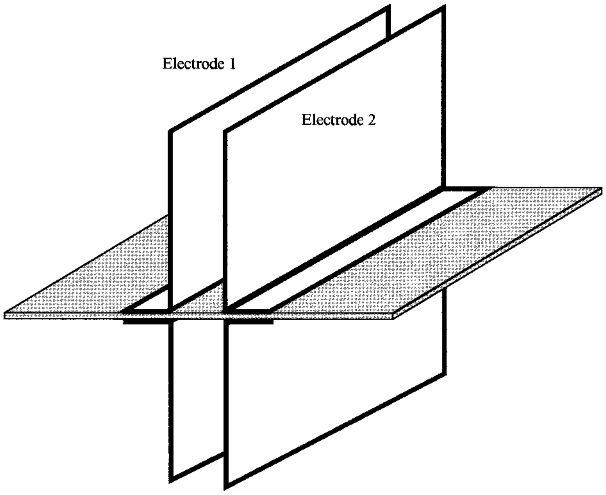

Figure 3 Sample mounted in parallel plate electrodes.

A third type of electrode was constructed to measure resistivity using Ohm's law for circuit theory, as illustrated in Figure 1. Two solid bars were used with square cross sectional dimensions of 0.32cm by 0.32cm. These measurements were compared to measurements acquired with L-shaped parallel plates and cylindrical electrodes using Ohm's law for field theory.

The material used for all resistivity measurements in this study was a 3M transparency film, IR 1140. Films were conditioned in the measurement chamber for 24 h before measurements were acquired. Resistivity was calculated using the average current from 100 readings measured at 10-min intervals. Temperature and RH in the chamber were recorded after each current measurement.

A coaxial wire was connected to one electrode and the voltage supply whereas the other wire was connected to the other electrode and an ammeter. A Keithley Model 617 ammeter capable of measuring current as small as 10^{-16}A was employed to measure current across samples. Samples, electrodes, and sample mounts were placed in a stainless steel measurement chamber and covered by another stainless steel plate to form an airtight Faraday cavity to shield the chamber from extraneous electric and magnetic fields. An Omega Model 219 transmitter was placed in the chamber to measure relative humidity (RH) and temperature. A saturated NaCl solution was placed in a pan located on the bottom of the cham-

Table I Surface Resistivity of IR1140 Film Measured by Ohm's Law for Field Theory Using Parallel Plate Electrodes and 100V Applied Voltage

Sample Length (cm)	Temperature (°C)	Relative Humidity (%)	Measured Current ($A \times 10^{-13}$)	Current CV (%)	Surface Resistivity (Ohm/square $\times 10^{14}$)
2.54	25.0	77.7	3.077	0.31	3.25
5.1	25.1	77.9	1.548	0.22	3.23

ber to control RH according to ASTM E104[5] so the chamber would provide a constant RH when temperature in the chamber was constant. The whole chamber was immersed in a large Plexiglas jar filled with water. Water temperature in the jar was controlled to ±1°C with an immersion heater and stirrer. In this way, air temperature in the measurement chamber was controlled to ±0.1°C and RH was controlled to ±0.2%. When the measurement chamber temperature was set to 25°C, ASTM E104 predicted it would have an RH of 75.3%.

RESULTS AND DISCUSSION

Relative humidity in the measurement chamber was 77.8 ± 0.2% when the chamber temperature was 25 ± 0.1°C. This RH value was 2.5% greater than that predicted by ASTM E104. The reason for this difference is unknown, although it may have resulted from a transmitter calibration error. However, the RH in the chamber was stable and consistent for all measurements. The coefficient of variation in RH was 0.24% for 100 readings measured at 10-min intervals over 1,000 min.

Table I shows surface resistivity values for IR1140 film measured by electric field theory using parallel plate electrodes separated by 2.54 cm and 5.1 cm. This table shows that values for the fundamental material constant, resistivity, were similar even though plate separation distance varied by a factor of 2.

Table II shows surface resistivity values for IR1140 film measured by Ohm's law for field theory using cylindrical electrodes with outer electrode diameters of 4.02 cm and 6.29 cm. This table shows that values for the fundamental material constant, resistivity, were similar even though outer electrode diameter varied by 50%.

Together, Tables I and II show that resistivity values based on Ohm's law for field theory are independent of both sample dimension and electrode structure.

In contrast to Tables I and II, Figure 4 shows surface resistivity values for IR1140 film measured by Ohm's law for circuit theory. Solid bar electrodes were separated by various distances so sample lengths ranged from 1.0 cm to 6.0 cm. This figure clearly shows that values of the material constant, resistivity, are not constant when measurements are based on Ohm's law for circuit theory. That is, resistivity values depend on the length of test samples. In addition, surface resistivities measured by Ohm's law for circuit theory (Fig.4) did not agree with those measured for the same samples by Ohm's law for field theory using either parallel plate electrodes (Table I) or cylindrical electrodes (Table II). This data illustrates

Table II Surface Resistivity of IR1140 Film Measured by Ohm's Law for Field Theory Using Cylindrical Electrodes and 100V Applied Voltage[a]

Outer Electrode Diameter (cm)	Temperature (°C)	Relative Humidity (%)	Measured Current ($A \times 10^{-12}$)	Current CV (%)	Surface Resistivity (Ohm/square $\times 10^{14}$)
4.02	25.0	77.5	1.026	0.16	3.32
6.29	25.1	77.8	0.8756	0.23	3.13

[a] Inner electrode diameter equals 0.64 cm.

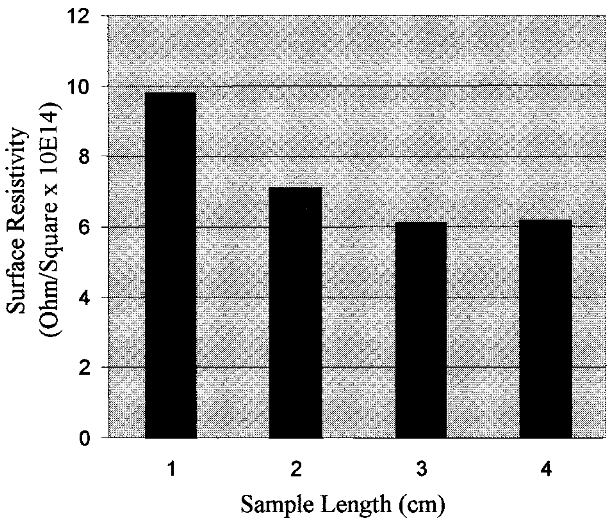

Figure 4 Surface resistivity of IR1140 Film measured by Ohm's Law for circuit theory using solid bar electrodes and 100V applied voltage.

that the importance of using Ohm's law for field theory cannot be overemphasized when measuring the surface resistivity of high-resistance materials.

Figure 5 shows the effect of temperature on surface resistivity. These measurements were acquired for IR1140 film using parallel plate electrodes separated by 5.1 cm and 100V applied voltage. The trend that was observed agrees with the electrical behavior generally expected from nonconductive materials. That is, increasing sample temperature decreased resistivity. Our data shows that a 10°C change in temperature changed sample resistivity by a factor of 6. This data clearly illustrates the importance of controlling temperature during resistivity measurements.

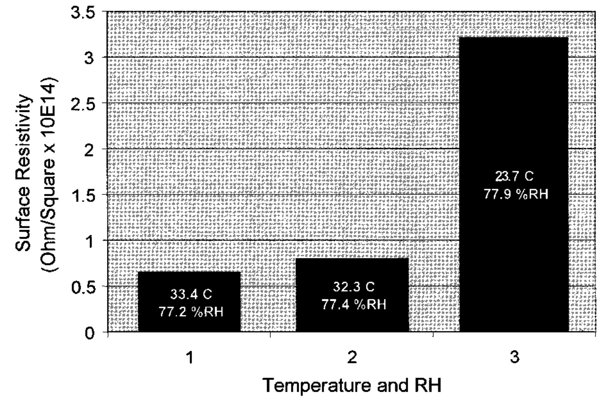

Figure 5 Effect of temperature and RH on surface resistivity of IR1140 Film measured using parallel plate electrodes and 100V applied voltage.

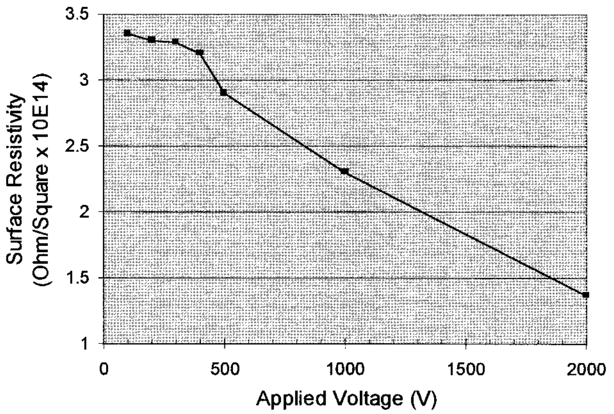

Figure 6 Effect of applied voltage on surface resistivity of IR1140 Film measured using cylindrical electrodes; inner electrode diameter equals 0.64 cm and outer electrode diameter equals 4.02 cm.

The influence of RH on resistivity of a particular sample depends on its water absorption capacity because current can be transmitted through water absorbed into the sample. Our film sample was substantially hydrophobic and the small changes in RH noted in Figure 5 would not be expected to influence resistivity significantly. However, controlling RH would generally be expected to improve resistivity measurements when hydrophilic samples are measured.

The influence of applied voltage on resistivity measurements was also investigated. Figure 6 shows resistivity measurements acquired for IR1140 film using cylindrical electrodes with an inner electrode diameter of 0.64 cm and an outer electrode diameter of 4.02 cm. This figure shows that relatively small increases in applied voltage decreased resistivity slightly. This data obeys the voltage-current curve described by Roth.[6] When applied voltage was increased significantly, however, the structure of the air between the electrodes changed and a corona discharge was produced. When this occurred, current drawn through the air (1μA to 1 mA) was substantially larger than current drawn across the sample. Consequently, resistivity measurements may be distorted and unreliable when high voltage is applied.

Finally, it is important to note that complete shielding of the measurement system is necessary to obtain measurements that are not disturbed by extraneous electric and magnetic fields. To avoid the influence of extraneous fields, it was necessary to shield the sample chamber with a Faraday cage as previously discussed. It also was necessary to use coaxial wire for all electrical connections.

CONCLUSIONS

The surface resistivity of high-resistance polymer films can be accurately measured if factors that affect measurements are well-controlled. These factors include (1) using Ohm's law for field theory to include the effects of electrode structure, (2) controlling RH and temperature in the measurement chamber, (3) limiting the level of applied voltage to maintain stable air structure between the electrodes, and (4) shielding the measuring system from extraneous electric and magnetic fields using a Faraday cage and coaxial wire.

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